



Work-In-Progress—A Virtual Reality Application for Learning Geometry

Yasamin Tahiri¹, Mutfried Hartmann², Thomas Borys³ and Daniela Maier⁴

^{1, 2, 3, 4} Pädagogische Hochschule Karlsruhe, Karlsruhe, Germany
yasamin.tahiri@ph-karlsruhe.de

Abstract. Dynamic geometry software makes it possible to manipulate and adapt geometric constructions. This enables an extended exploration of geometric relationships. This can improve the understanding of geometry. However, this software is often difficult to use and there are only few applications in the field of Mixed Reality, thus disregarding possible advantages such as an improved spatial perception in virtual space. For this reason, this Work-In-Progress presents a VR application that can be used to create geometric constructions. It is described how criteria of intuitive usage and functions of dynamic geometry software are used to simplify the usage and to use the advantages of Mixed Reality.

Keywords: Dynamic Geometry Software, Virtual Reality, Intuitive Usage.

1 Introduction

Dynamic geometry software (DGS) usually tries to cover as many sub-areas of geometry as possible within its functional scope. For example, in a DGS it is possible to examine the relationships between objects in more detail through the dynamic manipulation of objects [1]. The ability to dynamically manipulate objects and thus create a dynamic image [2] essentially distinguishes DGS from interactive geometry software. While a DGS is also interactive in principle, it is not possible to generate a dynamic image within interactive geometry software.

The interactivity within an interactive geometry software can be, for example, that the user is able to rotate a geometric body. Most geometry applications in the Mixed Reality (MR) spectrum are therefore interactive geometry software and not DGS. Using a DGS instead of interactive geometry software can make a difference in geometry understanding. Manipulating or building geometric constructions leads to a different learning effect than simply looking at geometric solids [3]. However, difficulties in dealing with DGS have also been noted [4, 5].

This paper presents a virtual reality (VR) application based on the basic functionality of DGS. The aim of this work is to show the usage differences between computer-assisted DGS and the VR application and to highlight the advantages of VR. For this purpose, the work has been divided into three sections. First, the theoretical basics are presented. Here, geometry software as well as the advantages and potentials of virtual reality for learning geometry are presented. In the second part, design principles as well as the VR application are presented and related to each other. The work concludes with a summary and an outlook on further research.

2 Theoretical Basis

2.1 Dynamic Geometry Software

There is both two-dimensional dynamic geometry software (2D DGS) and three-dimensional dynamic geometry software (3D DGS). 2D DGS are used for mathematical construction in the plane and outnumber 3D DGS. 3D DGS are used for mathematical construction in space. Compared to 2D DGS, 3D DGS can be used to build objects such as spheres or pyramids. It is also possible for dynamic geometry software to enable construction in both 2D and 3D. For example, the software Cinderella [6] and GeoGebra [7] can be mentioned here. While GeoGebra is

also available for mobile devices, among other things, most 2D and 3D DGS are usually available as desktop applications.

Compared to drawing with pencil and paper, dynamic geometry software has the advantage that by manipulating parameters of the construction, the construction does not have to be redrawn or created. This can save time, especially with complex designs. The log function of dynamic geometry software also makes it possible to retrace the executed steps of the construction in retrospect [6].

The number of dynamic geometry software in virtual space is very limited compared to two- and three-dimensional DGS. Construct3D is considered the first dynamic 3D geometry software [8] and enables the manipulation of geometric objects via Optical See-Through Head-Mounted Displays (OST-HMD) [9]. Direct manipulation is done via a stylus, while basic functions such as deleting are performed via the Personal Interaction Panel [9]. NeoTrie VR is another virtual 3D DGS application that allows 3D geometric objects and models to be created and dynamically manipulated [10].

2.2 Learning Geometry in VR

As mentioned earlier, DGS are characterized by the fact that they allow users to create dynamic images through the manipulation of objects. In virtual space, the successful manipulation of objects depends on several factors, including the features of the virtual environment, the chosen type of manipulation, and the characteristics of the individual users [11].

Within a virtual learning application, two main users emerge, namely the learner and the teacher. In the context of this article, only the learner will be discussed here. As a general characteristic of the learner, it can be assumed that most of them do not have any previous experience in using a virtual application, as virtual reality is not a common medium in households, despite the option of purchasing head-mounted displays on the market today. Accordingly, the design of a virtual application should support the natural interaction possibilities of humans to keep the cognitive load of learning the virtual application low [12]. Nevertheless, the use of the virtual application requires that the user's attention, spatial orientation, and motor skills are well enough developed to allow them to interact in the virtual environment [12]. If users already have experience in 2D or 3D DGS, it is likely that this experience can be transferred into a virtual application [13].

Learning and recognizing geometric relationships requires spatial understanding, which can, however, be improved using VR [14].

2.3 Advantages of VR

To be able to name the possible advantages of VR over computer-based dynamic geometry software, the weaknesses of computer-based DGS are described in more detail below.

The difficulties in using dynamic geometry software lie primarily in the control of its tools [4]. The term tool in the context of dynamic geometry software includes tools that are easy to understand, such as moving points, as well as tools that require several construction steps, such as cone cutting tools. Users are therefore encouraged to change tools regularly during construction, depending on the purpose. In practice, the tools are controlled via the computer mouse. With the help of VR, tools can be controlled via virtual hands, which could enable users to use the tools more intuitively. Similarly, it is possible that construction steps or even individual tools are grouped together in virtual space to also enable easier use.

Further difficulties in tool handling were found when switching from a 2D DGS to a 3D DGS [5]. Here, the use of a 3D DGS is found to be more difficult compared to a 2D DGS that was already being used [5]. Unlike in two-dimensional space, the work surface of a 3D DGS must be rotated by the user to set points accordingly or to manipulate them afterwards. This might seem unfamiliar to users, as interaction with the workspace in two-dimensional space is mainly limited to moving and zooming in/out.

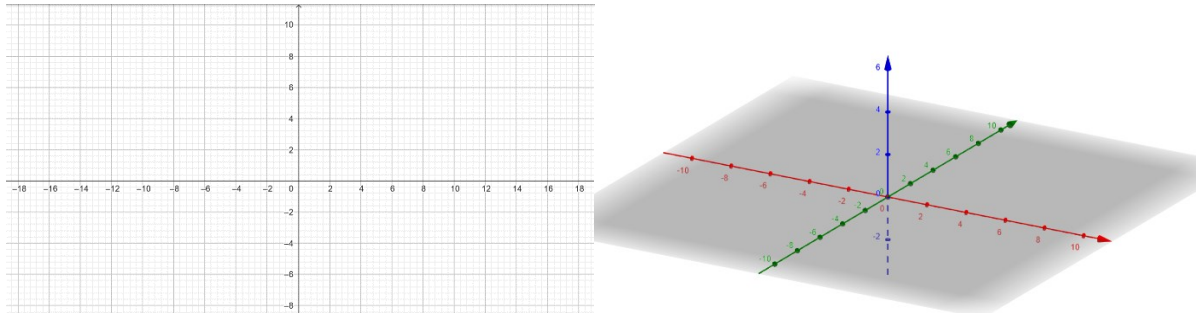


Fig. 1. 1. Workspace in two- and three-dimensional space (GeoGebra).

By using virtual space, the workspace can be controlled intuitively during construction, namely by rotating and moving one's own body in virtual space. This would be another possible advantage of VR over computer-based DGS.

To summarize, VR can be used to overcome the limitations of DGS, namely:

- The representation of 3D objects on desktop devices, and the resulting lack of depth understanding [14].
- The manipulation of objects, through simplified control with virtual hands and a reduction of construction steps.

3 Application

With the support of geometry experts, a VR application for the creation of geometric constructions is being developed. The VR application is developed using Unity3D and the plug-in "Auto Hand" as an interaction system for Oculus Quest 2. The development of the application is carried out in the context of regular usability feedback meetings, in which the experts test the status of the prototype. Suggestions for extending the range of functions consider potential work methods of users as well as criteria for intuitive usability.

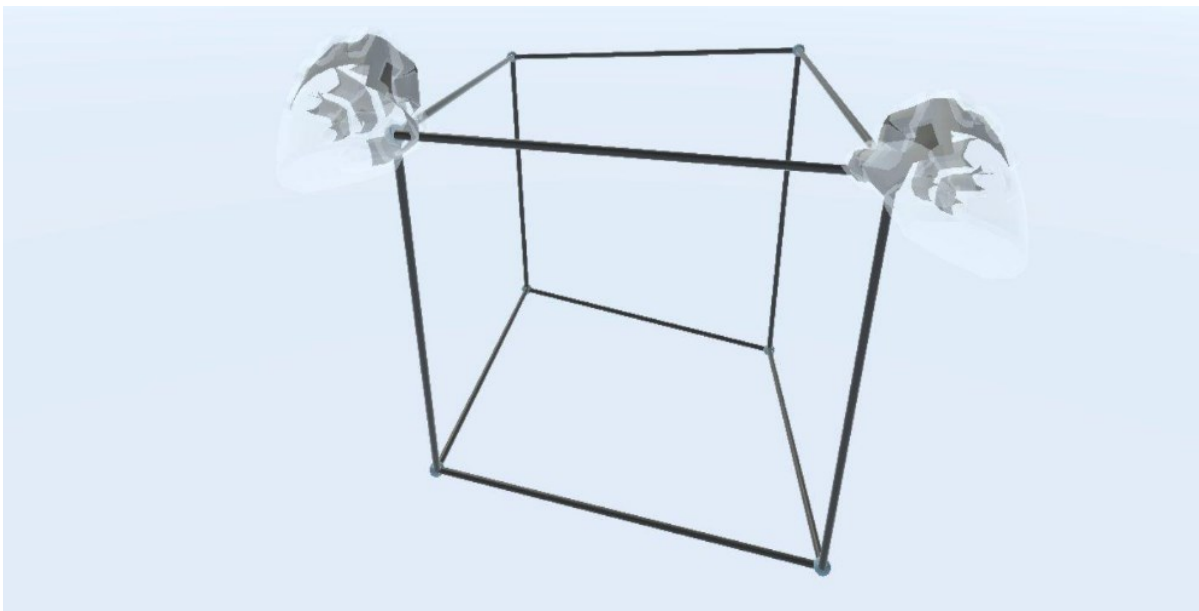


Fig. 2. Construction within the VR application.

3.1 Presentation of the Criteria

To take advantage of VR in an educational context, several criteria and design principles have been proposed [3], [15], [16]. The current state of the prototype development mainly considers the criteria of feedback and interaction / manipulation:

- 1) *Continuous feedback*: In the context of the prototype, feedback is currently not understood as feedback that evaluates the user's performance, but mainly as audiovisual or haptic feedback, such as the vibration

of the controller, during free construction. The feedback can vary both in duration and intensity. In the current version of the prototype, users mainly receive feedback through sounds and vibrations when performing actions.

- 2) *Manipulation and interaction:* With the controllers of Oculus Quest 2, it is possible for users to interact within the virtual world. It is possible to manipulate created objects as well as change the user's position and rotation using the analogue sticks. Interaction with objects within the prototype is mainly done with the virtual hands, e.g. by grasping. But interactions via the menu are also possible. By using the index finger, a laser pointer is generated that enables interaction with the menu.

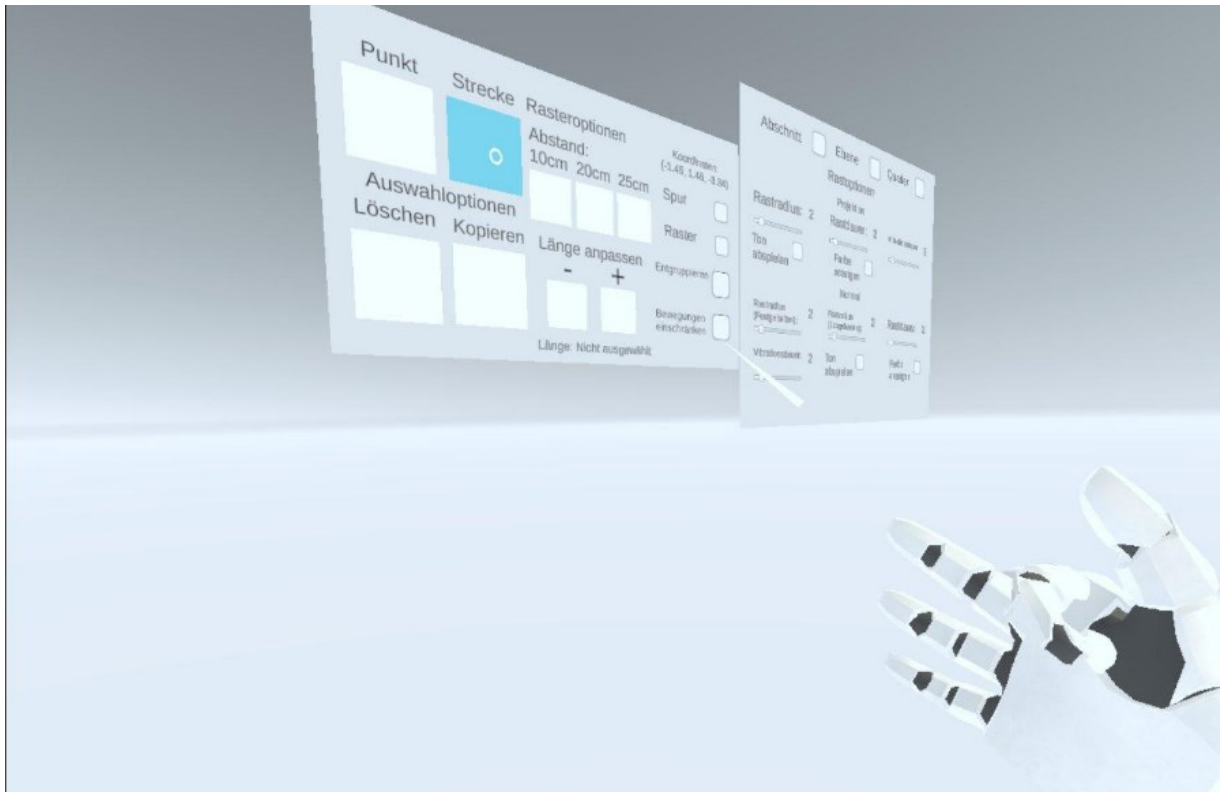


Fig. 3. Interaction with the menu.

3.2 The Line Tool

To create a line in a two- or three-dimensional dynamic geometry program, two points are always needed. Depending on the software, two points can be selected that are already present on the work surface, or the points are created together with the line. There are differences in the representation of the line if only one point is selected: In the first variant the length of the line is variably adjusted during creation. The length of the line is not fixed until the second point is selected. In the second variant, the line appears immediately after the second point is selected and is not visible beforehand.

Within the prototype, the line appears in the user's immediate field of vision when the corresponding tool is selected. In contrast to computer-based DGS, the line is already generated with its two points in a fixed size. Furthermore, it is possible not only to manipulate the position of the points, but also to grasp the line as an object, which is not possible with DGS. However, during the testing of the prototype, this type of manipulation appeared intuitive and can be compared in handling with the components of plug-in construction kits.

3.3 Magnetic Docking

A feature that most computer based DGSs have is the magnetic docking of points. For example, a point that is moved with the mouse pointer is docked along a grid or to the position of another point. Magnetic docking thus helps users realize during construction that their point is in the desired position.

This form of feedback is also integrated into the prototype, but - at least when docking to the position of other points - control of the point is not taken over. During the testing of the prototype, it felt unnatural to pull the point out of the hand or to control the hand with the point briefly through another source. Especially because it is not

recognizable whether the user wants the point to dock in this moment or not. Instead, an alternative step was introduced: If a point is close to another point, the transparency of the material is reduced for both points. Only when the point is released within this radius does the released point dock with the position of the other point. From this moment on, the positions of both points can be manipulated at the same time, as they are now connected to each other.

3.4 Further Features

In addition to the ability to create and edit points and lines, other features were included in the prototype. These were developed based on expert feedback about a potential user workflow. These functions include:

- Selection functions: Objects can be selected with a laser pointer without grasping them with the virtual hands. Selected objects then change their material color. This allows users to use functions such as deleting, duplicating, and adjusting the length of lines via menu buttons.
- Construction settings: By default, users can change both the angle and position of a point on a line when they grab it. However, it is also possible to switch off e.g., the change of the angle, if only a change of the length of the line is desired.

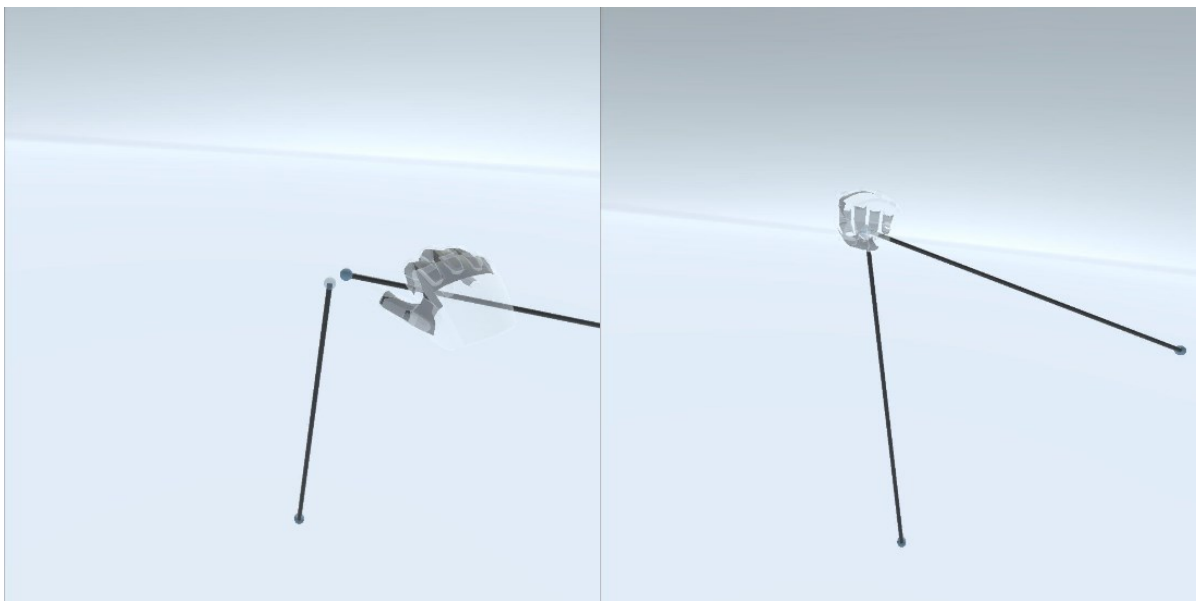


Fig. 4. Magnetic docking and manipulation of points.

4 Conclusion and Future Works

This paper presents the current state of the prototype of a DGS-based application. It was described how possible difficulties of computer-aided DGS, e.g., in the transition from 2D to 3D, can be solved by using VR. Using the example of the line tool, the transfer of functionalities into the virtual world and the adaptation regarding intuitive usage were described.

Currently, work is actively being carried out on various 3D grid alternatives for the prototype. In this way, it should be possible to align points and routes with integer coordinates. Various variants have already been designed and implemented for this purpose. Together with these variants and the functions presented in this work, empirical studies in the form of user tests are planned. In doing so, different research priorities will be set. On the one hand, the developed grid variants will be compared and evaluated. In this context, the users should also be given the opportunity to set feedback within the application, such as switching the sound on and off when docking. On the other hand, the users' preferences for the use of different visualizations should also be surveyed. This can include, for example, the display of different instructions and hints when (first) using the prototype.

Acknowledgements

This project is part of the “Qualitätsoffensive Lehrerbildung”, a joint initiative of the Federal Government and the Länder which aims to improve the quality of teacher training. The programme is funded by the Federal Ministry of Education and Research. The authors are responsible for the content of this publication.

References

1. Sarkar, P., Pillai, J.S., Gupta, A.: ScholAR: A collaborative learning experience for rural schools using augmented reality application. In: 2018 IEEE Tenth International Conference on Technology for Education (T4E). pp. 8–15. IEEE (2018)
2. Richter-Gebert, J., Kortenkamp, U.H.: Dynamische Geometrie: Grundlagen und Möglichkeiten. In: Proceedings of Nürnberger didactic Mathematics colloquium, [www document] http://www.cinderella.de/papers/DG_GM.pdf (2002)
3. Na, H.: Work-in-Progress-The Use of Plane-Detection Augmented Reality in Learning Geometry. In: 2021 7th International Conference of the Immersive Learning Research Network (iLRN). pp. 1–3. IEEE (2021)
4. Hohenwarter, J., Hohenwarter, M., Lavicza, Z.: Evaluating difficulty levels of dynamic geometry software tools to enhance teachers’ professional development. *International Journal for Technology in Mathematics Education*. 17, 127–134 (2010)
5. Hattermann, M.: Nutzerstudien zur Verwendung des Zugmodus bei Konstruktionsaufgaben in dynamischen Raumgeometriesystemen. *Journal für Mathematik-Didaktik*. 2, 209–236 (2013)
6. Kortenkamp, U.H.: Foundations of dynamic geometry, (1999)
7. Hohenwarter, M.: GeoGebra-ein Softwaresystem für dynamische Geometrie und Algebra der Ebene. (2002)
8. Kaufmann, H., Schmalstieg, D.: Designing immersive virtual reality for geometry education. In: Ieee virtual reality conference (vr 2006). pp. 51–58. IEEE (2006)
9. Kaufmann, H.: Collaborative augmented reality in education. Institute of Software Technology and Interactive Systems, Vienna University of Technology. 2–4 (2003)
10. Rodríguez, J.L., Morga, G., Cangas-Moldes, D.: Geometry teaching experience in virtual reality with NeoTrie VR. *Psychology, Society & Education*. 11, 355–366 (2019)
11. Poupyrev, I., Ichikawa, T.: Manipulating objects in virtual worlds: Categorization and empirical evaluation of interaction techniques. *Journal of Visual Languages & Computing*. 10, 19–35 (1999)
12. Bujak, K.R., Radu, I., Catrambone, R., MacIntyre, B., Zheng, R., Golubski, G.: A psychological perspective on augmented reality in the mathematics classroom. *Computers & Education*. 68, 536–544 (2013)
13. Kaufmann, H., Dünser, A.: Summary of usability evaluations of an educational augmented reality application. In: Virtual Reality: Second International Conference, ICVR 2007, Held as part of HCI International 2007, Beijing, China, July 22–27, 2007. Proceedings 2. pp. 660–669. Springer (2007)
14. Lai, C., McMahan, R.P., Kitagawa, M., Connolly, I.: Geometry explorer: facilitating geometry education with virtual reality. In: Virtual, Augmented and Mixed Reality: 8th International Conference, VAMR 2016, Held as Part of HCI International 2016, Toronto, Canada, July 17–22, 2016. Proceedings 8. pp. 702–713. Springer (2016)
15. Johnson-Glenberg, M.C., Ly, V., Su, M., Zavala, R.N., Bartolomeo, H., Kalina, E.: Embodied agentic STEM education: effects of 3D VR compared to 2D PC. In: 2020 6th international conference of the immersive learning research network (ilrn). pp. 24–30. IEEE (2020)
16. Tahiri, Y., Florian, L., Hartmann, M.: Intuitive Werkzeuge gestalten: Designprinzipien zur Entwicklung einer dynamischen Geometriesoftware im virtuellen Raum. *MedienPädagogik: Zeitschrift für Theorie und Praxis der Medienbildung*. 47, 94–117 (2022)