



Exploring Teachers' Perspectives on the Affordances and Constraints of Augmented Reality in Geometry Education

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Abstract. This paper identifies the affordances and constraints teachers experienced using an augmented reality (AR)-based geometry learning simulation—GeoGebra-MR. The study operationalizes these constructs within the context of AR-based learning and introduces a subcategorization framework. AR-based dynamic geometry software (DGS) facilitates geometric reasoning by visualizing 3D geometric objects and providing real-time interactive feedback in a collaborative networked space. Researchers developed a coding scheme and decision tree to systematically classify usability issues and underlying needs to improve the design of the platform to make it more intuitive. Video recordings of K-12 geometry teachers' geometric reasoning using the AR environment along with post-interviews highlighted several issues related to sensory, physical, cognitive, and functional affordances, and the corresponding needs linked to physical, logical, and cultural constraints. This study provides design recommendations for optimizing AR-based geometry learning environments, emphasizing that usability challenges are not solely technological but also shaped by teachers' perceived affordances and their attunement to constraints. By highlighting these factors, this paper offers valuable insights for researchers and educators seeking to enhance the design and implementation of AR in mathematics education.

Keywords: Augmented Reality (AR), Affordances, Constraints, Usability Issues (UX), Geometry Education, Teacher Perspectives, Collaborative Learning, Dynamic Geometry Software (DGS).

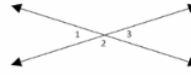
1 Introduction

Geometry curricula are often presented and taught through disembodied formalisms, such as equations and symbols [1, 2]. Kelsey and colleagues [1] demonstrated how these instructional practices can lead to poor geometric reasoning, using a widely taught and applied concept in geometry education—the two-column proof for the *Vertical Angles Theorem* (Fig. 1).

Example Proof of Vertical Angles Theorem

Given: $\angle 1$ and $\angle 3$ are vertical angles.

Prove: $\angle 1 \cong \angle 3$



Statements	Reasons
1. $\angle 1$ and $\angle 3$ are vertical angles.	1. Given.
2. $\angle 1$ and $\angle 2$ are supplementary $\angle 2$ and $\angle 3$ are supplementary.	2. Angles that form a linear pair are supplementary.
3. $m\angle 1 + m\angle 2 = 180$ $m\angle 2 + m\angle 3 = 180$	3. The sum of the measures of supplementary angles is 180.
4. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	4. Transitive Property of Equality
5. $m\angle 1 = m\angle 3$	5. Subtraction Property of Equality
6. $\angle 1 \cong \angle 3$	6. Angles with the same measure are congruent.

Fig. 1. Two-column geometry proof for the *Vertical Angles theorem*. (Reprinted from Kelsey and colleagues [1]).

They identified several key obstacles that novices face in understanding the proof. Highly formalized terms, such as $\angle 1$ and $m\angle 1$, may be unfamiliar to students. Implicit assumptions also create confusion, such as the distinction between $m\angle 1$ and $m\angle 2$ as measurable quantities that can be arithmetically added, while $\angle 1$ and $\angle 2$ serve only as labels and cannot be manipulated. Additionally, operations like the *Transitive Property of Equality* (Line 4 in Fig. 1) are presented as static statements, making the reasoning behind them less transparent. This example highlights how most students experience geometry as an amodal subject, disconnected from sensorimotor systems, which leads to a superficial and often meaningless understanding of geometric concepts [1].

In response to these issues in geometry instruction, there is growing appreciation for the role of early sensorimotor experiences in facilitating a grounded understanding of abstract formalisms [2]. Augmented reality (AR) technology provides opportunities to simulate these sensorimotor experiences in K-12 classrooms, allowing teachers to help students map abstract formalisms onto meaningful, embodied experiences. Studies have demonstrated that successful mathematical problem-solving often requires physical interaction with mathematical objects—such as touching, rearranging, or grouping them [3, 4].

It is imperative that teachers understand how best to integrate AR into their curriculum. As a result, this imperative has become a focus for researchers and designers to address issues with its implementation. Since teachers play a key role in the successful integration and acceptance of new technologies in classroom education [5], developers must include teachers and their perspectives as part of the design process to ensure the technology's effectiveness as a pedagogical tool for instruction. Teachers' curricular expertise and experience provide valuable insights not only into the accuracy of the content and the platform's usability, but also into how AR-based simulations like GeoGebra-MR can complement classroom activities compared to previous technological tools like GeoGebra-Classic, a web-based 2-dimensional (2D) version of the software (Fig. 2).



Fig. 2. Geometry education with real-world objects vs. GeoGebra-MR vs. GeoGebra-Classic.

Well-designed environments both *afford* and *constrain* users' abilities to intuitively perceive and interact with functionalities in a way that is easily understood [6, 7]. When features of AR design (e.g., shape, color, sound, menus, buttons, gestural mechanics, scaffolds, feedback, etc.) do not align with users' expectations for intuitive interactions, usability issues arise which can frustrate users, fostering negative attitudes toward the technology and ultimately hindering the learning process. Consequently, the design of human-computer interactions (HCI) necessitates inspecting, classifying, and analyzing the affordances and constraints of a technology as foundational for its usability [8, 9].

It is crucial that discussions of affordances and constraints in HCI are applied with proper clarification for how they were originally defined by both Gibson [10] and Norman [11] [8]. For instance, many discussions on the affordances of computer system designs have mistakenly conflated system functions with affordances [8, 12]. That is, there is a need for clearer distinctions beyond user interfaces and addressing overall system designs. To do so, we revisit the use of affordance (and constraints) and refine their definitions [8, 11, 12] in the context of AR to comprehensively discuss its design and deployment [6].

As a relatively new and proprietary technology, affordances (and constraints) of AR are neither equivalent to those evoked by traditional interface metaphors in web-based HCI nor analogous to those found in the real world that they attempt to emulate. As a result, the conventions of real-world physical rules or common web-based interface logic are both insufficient to perceive and interact with AR-based virtual worlds. Thus, the challenges in understanding the affordances of virtual environments underscores the need for tailoring HCI specifically to AR-based learning environments. In this study, we aim to investigate teachers' perspectives on AR-based geometry education by identifying the usability issues they encounter, utilizing a refined definition of affordances specifically developed for AR environments.

2 Theoretical Framework

2.1 Affordances and Constraints

Affordances. Gibson's [10] concept of affordances is an interactionist view of perception and action that focuses on the dynamic interactions between agents and their environments. He argued that many questions about how learners construct information could be solved by examining the sources of information and the environment they are situated in to understand how learners use them in their activities. Norman [11] extended and refined the concept of affordance, highlighting common misinterpretations, particularly the confusion between "perceived" affordances and "real" affordances. Perceived affordances have a visible presence that provides clues (i.e., meta-evaluative recognition) for their operation, while real affordances refer to properties inherent in the environment and possible actions that users may not be aware of (i.e., visible clues) or recognize. By distinguishing between perceived and real affordances, Norman emphasized that designers should make the desired, relevant actions of real affordance readily perceivable. For the current research, we limit the definition of affordances to perceived affordances since we cannot make assumptions about users' actions in the absence of visible clues. Our focus is restricted to moments when users' intuitive understanding of features (and their affordances) fails, making their presence more evident—specifically, when we can observe issues with teachers using GeoGebra-MR. Nonetheless, both Gibson's and Norman's articulation of affordances still lack specificity for identifying usability issues in HCI. Fortunately, Hartson [8] extended the concept of affordance into four sub-categories that make it useful for addressing the issues that teachers encountered using GeoGebra-MR.

Specifically, Hartson [8] categorized four types of affordances based on their role in supporting users during interaction: (1) *cognitive affordance* indicates design features that help users with their cognitive actions such as understanding the meaning of sensorial cues or system outputs; (2) *physical affordance* refers to design features that help users with their physical actions such as manipulating artifacts quickly, easily, and reliably; (3) *sensory affordance* refers to design features that help users with their sensory actions such as sensing or discerning the sensorial cues, system outputs, or artifacts for manipulation; and (4) *functional affordance* indicates design features that help users to achieve goals through task performance. The present study applied these four types of affordances for systematically and thoroughly coding usability issues and regarding needs.

Constraints. Constraints are the actionable properties that users attune to limit their interactions. It is important to consider that constraints do not necessarily imply challenges or negative experiences for users. In fact, the concepts of affordance and constraints are always intertwined. Gibson [10] defined affordances for agents as conditions in the environment for constraints to which the agent is attuned. Several researchers have considered attunement to constraints as a basis for making inferences about interaction with properties of the environment [13]. For example, when a learner is attuned to the logic of AR—such as understanding that they can only manipulate holographic properties when their color changes—they stop attempting to interact with random elements in the virtual world and instead infer whether the properties afford manipulation. Lack of attunement to constraints leads to low user ability to utilize affordances properly.

Norman's [11] extension of Gibson's [10] work also categorized three kinds of behavioral constraints: *physical*, *logical*, and *cultural* constraints. Although these categories are not necessarily mutually exclusive, this framework establishes operational criteria between the categories that offers a clearer distinction of our coding scheme for the current evaluation of the AR-based environment. Thus, we operationalize these three types as follows: (1)

physical constraints refer to immutable constraints that users cannot violate or ignore; (2) *logical constraints* indicate constraints that are inherent to the logic and mechanics of the AR environment, which may differ from the physical laws and intuitions of the real world. For example, if the logic of the AR environment, although different from real-world physics (which are known as physical constraints), is understandable and transferable to other interactions within the same AR context, it is categorized as a logical constraint; and (3) *cultural constraints* arise from established norms in the user communities. These norms are carried over from previous iterations of earlier version of the software (i.e., GeoGebra-Classic) or with other AR environments. Cultural constraints are shaped by the established norms and or conventions of the AR setup and can also be influenced by individuals' experiences with similar virtual environments, other geometry education technologies, and teaching practices. For example, if a constraint within the AR environment cannot be easily understood or applied to other interactions within that environment (which are known as logical constraints), it is considered a cultural constraint.

2.2 Affordances & Constraints of AR-based Geometry Education

The affordances of AR environments differ fundamentally from those in traditional desktop-based learning environments and other educational technologies [6, 14]. In mathematics learning, Shaffer and Kaput [15] suggested that computational media introduces a new stage of cognitive development by enabling the externalization of mental processes and creating novel representation forms. Using AR technology for GeoGebra allows learners to externalize inference processes through the physical manipulation of geometric objects, along with external representations of measurements (such as lengths, angles, and areas) that can be tested, modified, and discussed. This capacity facilitates users' abilities to articulate and reflect on mental activities involved in mathematical reasoning. Furthermore, the novel representational power of AR—such as manipulatable ideal mathematical objects in dynamic geometry environments—allow users to approach abstract mathematical problems more concretely. Body-controlled input devices in AR provide opportunities for more concrete and intuitive psychomotor learning experiences [16-18].

To fully integrate these new affordances in geometry education, it is crucial to understand the practical needs of the classroom from teachers' perspectives. While some studies have begun exploring teachers' views on the limitations and opportunities of AR [14, 19], the current research will contribute to following question: How do teachers intuitively interact with these novel affordances? What are the constraints? What usability issues arise?

3 Methods

3.1 Participants

Geometry teachers ($N = 6$) from a high school in the Midwest of the United States were invited to a laboratory space at a large research university in the Midwest. They used GeoGebra-MR (Fig. 3), an AR-based simulation designed for geometric reasoning. This study adhered to all ethical guidelines for human subjects research as outlined in the Common Rule and received full approval from the university's Institutional Review Board (IRB).

3.2 Materials

GeoGebra-MR includes various geometry objects that users can manipulate using hand gestures (see Fig. 3). Users can measure or disassemble (e.g., unfolding a cylinder) the 3D objects using the virtual dashboard interface in GeoGebra-MR.

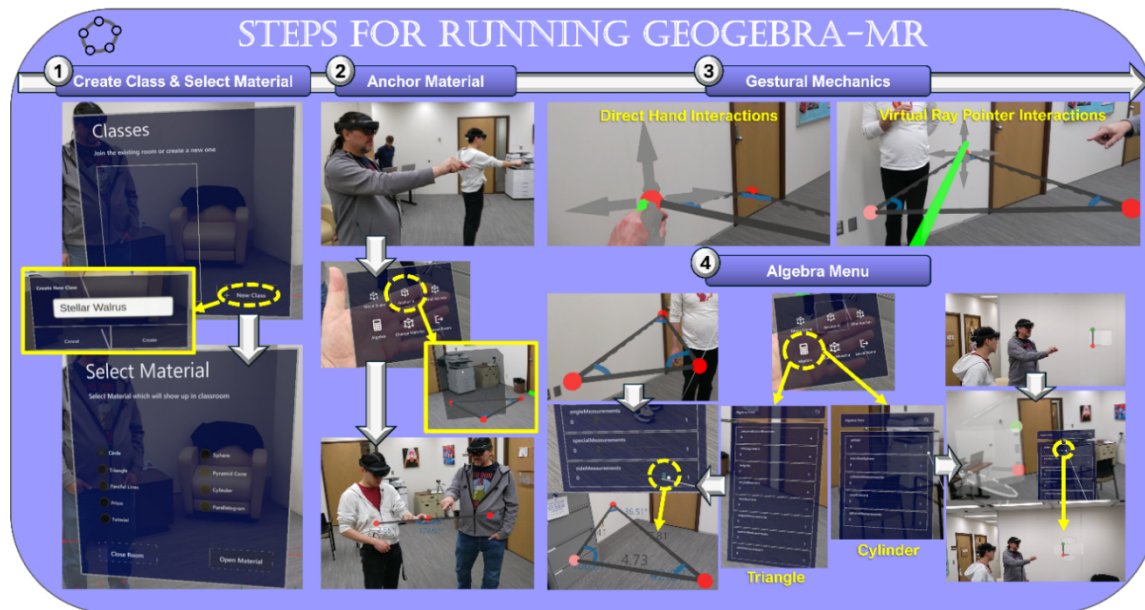


Fig. 3. Flow of GeoGebra-MR: (1) Users create a new class and select the material for the class; (2) Users anchor the material to calibrate the position of the holographic object so it appears the same for all users; (3) Users can manipulate the objects using either their hand or a virtual ray pointer; (4) The Algebra menu allows users to toggle features on and off (e.g., side and angle measurements, or disassemble the object, etc.).

3.3 Procedure

To encourage natural interaction and collaboration, teachers were paired into three dyads. Pair work facilitates learning through gestures, representations, and shared reasoning while allowing participants to explore multiple perspectives and extend their cognitive resources within the AR environment.

Each dyad wore an AR headset (Microsoft HoloLens 2) and was accompanied by a researcher wearing a third headset to proctor the session. They were tasked with solving two geometry conjectures developed by Nathan and colleagues [20], which were selected from secondary mathematics textbooks and validated by mathematicians and math educators (see Fig. 4). Teachers' participation was recorded by logging headsets into web-based portals on laptops. Upon completion, video files were saved to a secure database, after which teachers participated in semi-structured post-interviews that were subsequently transcribed for analysis.

Data were analyzed using a developed coding scheme and decision tree (as described in Section 3.4.), with discrepancies in coding resolved through iterative discussions. The framework was developed collaboratively. As a PhD student in Learning Sciences with six years of experience in AR development, I collaborated with a tenured professor in Learning Sciences to establish the framework. The coding process underwent cross-validation to ensure reliability, followed by final revisions with two graduate students and a postdoctoral researcher specializing in Learning Sciences.

Conjecture	GeoGebra-Classic	GeoGebra-AR
The sum of the lengths of two sides of a triangle is always greater than the length of the third side.		
Given a cylinder with radius r and height h , this cylinder can be unrolled to include a rectangle with length h and width $2\pi r$.		

Fig. 4. Geometric conjectures and diagrams in GeoGebra-Classic and GeoGebra-MR.

3.4 Coding Scheme and Coding Decision Tree

The video data of participants' interactions with GeoGebra-MR were analyzed using a coding scheme that identified the affordances and constraints that teachers encountered (adapted and revised from the definitions by Hartson [8] and Norman [11]) (see Fig. 5).

Affordance	Decision prompts	Constraints	Decision prompts
Cognitive	Design feature that helps users <i>in understanding the meaning</i> of the sensorial cues or system outputs	Physical	<i>Immutable</i> characteristic (cannot be violated or ignored)
Physical	Design feature that helps users <i>in manipulating</i> artifacts quickly, easily, and reliably	Logical	Reasoning with a conceptual model (<i>transferable logic</i> of the AR environment)
Sensory	Design feature that helps users <i>in sensing, discerning</i> the sensorial cues, system outputs, or artifact for manipulation	Cultural	Arbitrary reasoning (arising from <i>consistent prior experiences</i>)
Functional	Design feature that helps users <i>in achieving user goals</i> through task performance		

Fig. 5. Coding scheme for affordance types (*left*) and constraint types (*right*).

The coding process used a coding decision tree to determine the affordances and constraints of features and attributes of GeoGebra-MR (see Fig. 6). It was developed and refined through iterations of data to provide a systematic approach to this analysis. Our analysis focused on usability issues and needs—defined as instances where users were unable to perceive, understand, and interact with design features intuitively—to examine only observable affordances and constraints in GeoGebra-MR. As aforementioned, usability issues were categorized into one of the four affordance types (i.e., sensory, physical, cognitive, and functional). Additionally, the underlying needs for each usability issue were further classified into UX design problems, technical challenges, and constraints (i.e., physical, logical, and cultural). It is important to note that in the process of coding constraints, we denoted instances in which teachers' interaction with features of GeoGebra-MR were misaligned (evidence of the lack of attunement).

The narrative explanation for Fig. 6 is as follows. Usability problems were categorized based on the type of affordance they involved. First, issues with sensing or discerning sensory cues or system outputs were categorized as sensory affordance problems. These were further classified into the need for training on logical constraints, UX design issues, or technical challenges, depending on the underlying causes. Second, if users could sense or discern sensory cues but encountered interaction difficulties, the issue was classified as either a physical or cognitive affordance problem. Physical affordance issues typically involved difficulties in manipulating AR objects and were further divided into the need for training on logical constraints, UX design issues, or technical challenges. On the other hand, cognitive affordance issues arose when users struggled to understand the meaning of system resources, which were also categorized into the need for training on cultural constraints, UX design issues, or technical challenges. Lastly, problems related to solving the geometry conjecture were classified as functional affordance issues, often accompanied by UX design recommendations. If none of these categories applied, the issue remained unspecified based on the current framework.

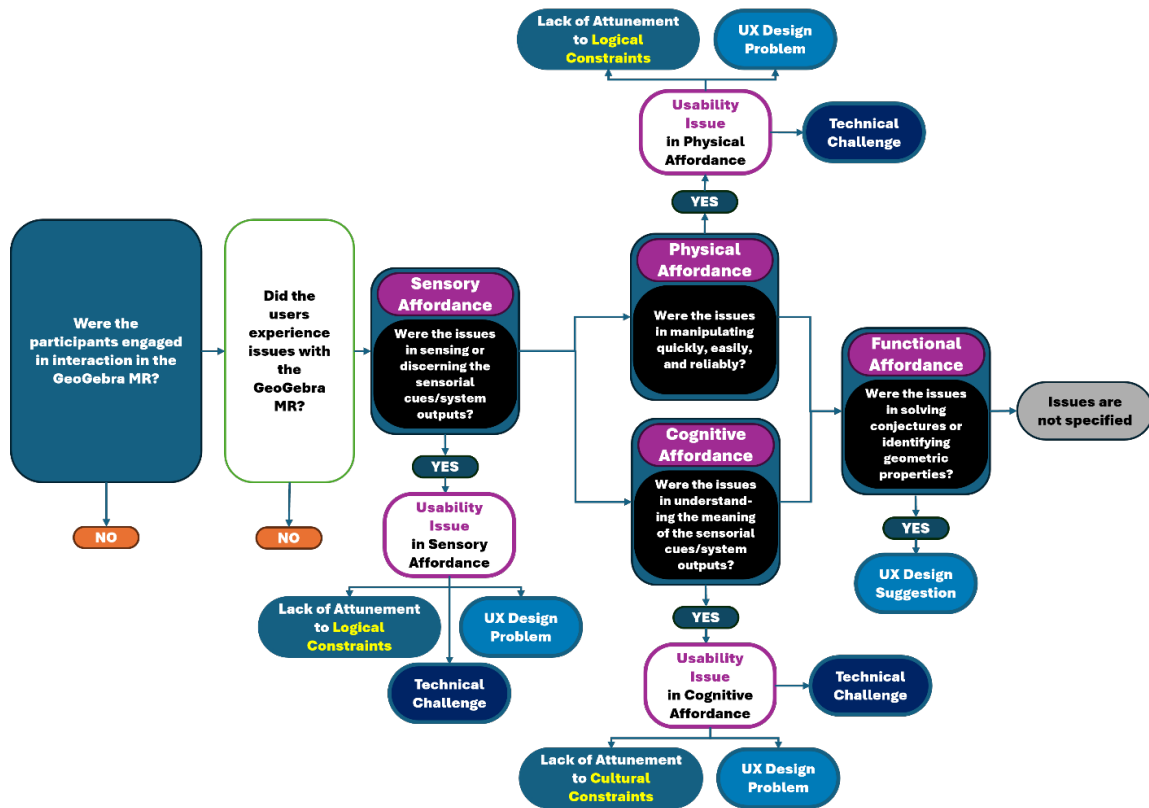


Fig. 6. Coding decision tree for usability issues and needs regarding affordances and constraints.

4 Results

4.1 Sensory Affordance

While many issues were technical, some arose from teachers' lack of attunement to logical constraints. Two main issues were identified. First, teachers often couldn't sense the AR objects due to the restricted field of view (FOV) in AR (*technical challenge*) and their *lack of attunement to the logical constraints* of AR's FOV (see Fig. 7 (left)). Second, teachers struggled to discern who was controlling the objects when multiple people tried to manipulate the same object, since the system output failed to differentiate between users, indicating a need for improvements in *UX design* (see Fig. 7 (right)).



Fig. 7. Restricted FOV in AR (left) and unclear attribution of control (right).

4.2 Physical Affordance

Nearly half (41.1%) of the usability issues were identified as physical affordance problems. Most of those issues (86.4%) stemmed from teachers' tendency to rely on physical constraints, leading them to interact with AR as they would the physical objects in the real world. For example, teachers couldn't successfully grab or manipulate AR objects since the AR device only detected predefined gestures (with limited variability), such as pinching or clasping (*technical challenge*), and teachers were *not well attuned to the logical constraints* of physical interaction

with 3D objects (see Fig. 8 (left)). Some issues were classified as *UX design problems*. For example, teachers struggled with fine manipulation, such as selecting menu sliders to turn features on and off or selecting hotspots to resize 3D objects (see Fig. 8 (right)).

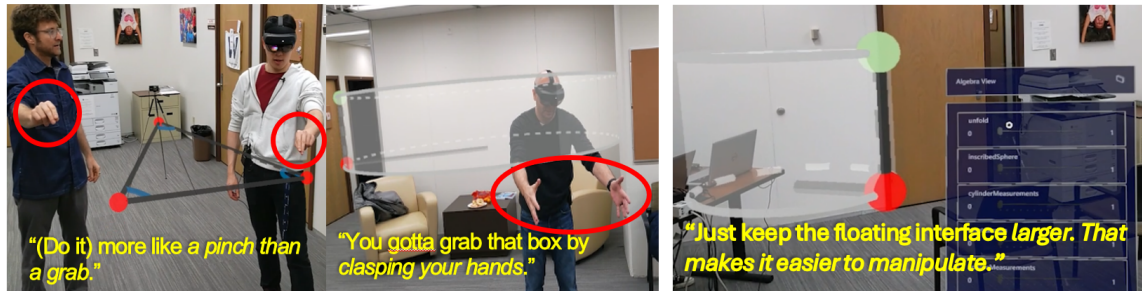


Fig. 8. Limited gesture recognition in AR (left) and fine manipulation difficulty (right).

4.3 Cognitive Affordance

The usability issues identified as cognitive affordances were primarily due to teachers' *lack of attunement to the cultural constraints* (62.5%), as they mistakenly conflated the constraints of GeoGebra-Classic—the previous web-based version—with GeoGebra-MR. They assumed that GeoGebra-MR had all the functions of GeoGebra-Classic and frequently asked how to activate features that are not available in GeoGebra-MR (e.g., labeling, locking and fixing objects, undo function, etc.). Moreover, the novel symbolic features of GeoGebra-MR also caused confusion. For instance, in GeoGebra-MR, two types of symbolic rays extended from users' fingers serve different functions. In Fig. 9 (left), although both rays seem to point at the same virtual dashboard interface, the white ray interacts with the interface, while the green ray interacts with the geometric object behind the menu. Additionally, inconsistent feedback from the AR system (*a technical challenge*) hindered users' attunement of GeoGebra-MR's cultural constraints. For instance, teachers and researchers mistakenly believed they needed to anchor 3D objects twice (see Fig. 9 (right)) because the system inconsistently provided feedback on their inputs.

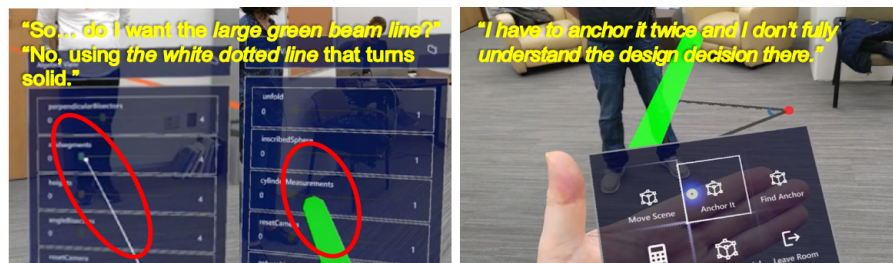


Fig. 9. Unfamiliarity with symbolic features (left) and inconsistent feedback (right)

4.4 Functional Affordance

Most of the *UX design suggestions* for enhancing functional affordances were made when teachers encountered conflicts between the cultural constraints of GeoGebra-MR and GeoGebra-Classic. Features from GeoGebra-Classic, such as algebra functions and labeling functions, were requested and suggested for integration into GeoGebra-MR. Teachers also commonly emphasized the importance of balancing playfulness with learning and proposed the need for a structured AR curriculum with goal-based activities.

5 Discussion

This study offers valuable insights for both practitioners who design and develop AR-based learning environments and researchers interested in exploring the pedagogical integration of AR in education. It introduces a novel coding scheme and decision tree for identifying usability issues and needs based on affordances and constraints in AR, addressing a need in the current literature regarding the definition and identification of affordances and constraints in AR. Additionally, these findings provide evidence-based design principles for developing AR-based systems

for learning, incorporating teachers' perspectives as experts in content and pedagogy, which are invaluable for successfully integrating technological tools into classroom activities.

Importantly, this study primarily examines teachers' perceived affordances to gain deeper insight into their perspectives—factors that researchers or practitioners might overlook. While usability issues are often framed from technological and design perspectives, our findings highlight that these issues are mediated by learners' attunement to various constraints. Notably, usability concerns should not be solely defined by technological limitations; rather, they must be understood in relation to users' expectations and perceived constraints.

The results reveal that sensory, physical, and cognitive affordance issues identified in this study indicate a dual need: first, for *technical* and *UX design* improvements, such as expanding the FOV, introducing a wider range of hand gestures, ensuring consistent feedback, and integrating features from GeoGebra-Classic; and second, for better *attunement of users to constraints*. This distinction is crucial, as usability issues may not always stem from system limitations but from how users perceive and engage with the technology. For instance, teachers did not perceive the absence of tactile-sensory feedback in AR as a usability issue, as they primarily viewed AR as a tool for visualizing geometric representations rather than manipulating real geometric objects.

Moreover, the study underscores that different affordance types require distinct attunement processes. The results show that physical affordance issues were primarily related to a lack of understanding of how physical interaction in AR (*logical constraints*) differs from real-world physical interactions (*physical constraints*). On the other hand, cognitive affordance issues arose from a misunderstanding of how GeoGebra-MR's features (*cultural constraints*) differ from those of GeoGebra-Classic (*misapplied cultural constraints*). Moreover, in the process of attunement to constraints, we found that attunement to symbolic constraints (e.g., different functions of two symbolic ray types) were relatively more adaptable and changeable for teachers than physical constraints (e.g., pinching and clasping hand gestures to grab 3D objects). These findings provide valuable guidance for developing targeted constraint training modules and addressing learners' challenges more effectively.

Limitations of this study did not account for participants' characteristics that could have affected their attunement to the constraints of AR. Specifically, we did not analyze participants' familiarity with AR technology or GeoGebra-MR, which limits the validity of our conclusions about the types of constraints that led to various usability issues. Future research should collect data on participants' *a priori* characteristics, such as AR experience, to better investigate usability issues specific to GeoGebra-MR.

Moreover, the coding scheme and decision tree developed in this study were specific to our evaluation of teachers' usage of GeoGebra-MR, which may limit its generalizability to other contexts. Development and application of a decision tree to assist in the design of other AR-based learning environments may require a reprioritization of certain affordances and constraints. For instance, GeoGebra-MR offers proprietary metaphorical features, such as floating menus and interactive feedback that aligns with logical rather than physical constraints. However, in other AR systems designed to prioritize realistic, authentic scenarios, physical constraints might take precedence over logical ones. In such cases, the need for logical constraints could be replaced by physical constraints in the coding decision tree. Additionally, validating the coding scheme and decision tree and establishing reliability will require a larger study with more observations that might offer insights for development and design that generalize across various AR-based learning platforms.

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